LIFT-OFF FELLOWSHIP REPORT

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I was supported by an AustMS Lift-Off Fellowship from November 2016 until my assumption of a Postdoctoral Research Fellowship at Macquarie University in May 2017. My research in this period was concerned with a certain problem in three-dimensional category theory which I encountered when writing my PhD thesis [1]. This research has, at the time of writing, resulted in two papers (one published [2], one under review [3]) and other work in progress (see for example [4]).

The main approach to studying low-dimensional higher category theory is via coherence theorems, which show that the general "fully weak" structures of this theory can be modelled by more manageable "strict" structures. For example, the fundamental coherence theorem of three-dimensional category theory [5] states that every tricategory (a "fully weak" three-dimensional category) is triequivalent to a **Gray**-category (a category "strictly" enriched over the monoidal category **Gray** of 2-categories equipped with the symmetric Gray tensor product).

In my PhD thesis, I showed moreover that much of the *category theory* of tricategories can be modelled by constructions of **Gray**-enriched category theory. However, I encountered a basic obstruction to giving a *purely* **Gray**-enriched model for the complete theory: the strictification functor, which assigns to each bicategory a biequivalent 2-category and thus relates the bases of tricategory theory and **Gray**-enriched category theory, cannot be made into a **Gray**-enriched functor. This obstruction can be understood in the context of enriched homotopy theory as a consequence of the fact that not every 2-category is cofibrant [6], for if a monoidal model category $\mathcal{V}$ (with cofibrant unit object) has a $\mathcal{V}$-enriched cofibrant replacement comonad, then every object of $\mathcal{V}$ must be cofibrant [7].

Following the completion of my PhD thesis, I set out to understand how best to accommodate this obstruction, with the goal of completely modelling the category theory of tricategories by homotopy coherent enriched category theory. An analysis of the fine structure of the strictification adjunction (carried out in [3]) suggested the concept of a *locally weak comonad*. I found that the natural context in which to prove the basic results on locally weak comonads that I needed was the theory of *skew-enriched categories*, a "skew" generalisation of the notion of enriched category which I introduced in [2]. I also investigated the general problem of what structure ought to be possessed by the cofibrant replacement comonad of a model category enriched over a monoidal model category in which not every object is cofibrant, which led me to revise the notions of monoidal and enriched algebraic weak factorisation systems [8] and show that their cofibrant replacement comonads are monoidal comonads and locally weak comonads respectively (see [4]).

At the time of writing, my more refined solution to this problem is to work not only over the base monoidal model category **Gray** of 2-categories, but also over the base monoidal model category of *algebraically cofibrant 2-categories*, in which every object is cofibrant, and whose full subcategory of fibrant objects is equivalent to the category of bicategories and normal pseudofunctors, and over which the (normal) strictification adjunction is enriched.

References

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