A modular proof of the straightening theorem

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The development of ∞-category theory in Lurie's book *Higher Topos Theory* [4] is founded on a series of rectification theorems, the first of which is the (unmarked) Straightening Theorem. This theorem states that, for each simplicial set A, the straightening–unstraightening adjunction is a Quillen equivalence between the simplicial presheaf category $[\mathfrak{C}(A)^{\mathrm{op}}, \mathbf{sSet}]$ equipped with the projective Kan model structure and the slice category $\mathbf{sSet}/A$ equipped with the contravariant model structure (whose fibrant objects are the right fibrations over A). Lurie's proof of this theorem is notoriously difficult; alternative proofs – substantially different from Lurie's proof and from each other – have since been given by Stevenson [5] and by Heuts and Moerdijk [2].

In this talk, I will present a new, simple proof of the Straightening Theorem. This proof is based on an idea which may be found in §51 of Joyal's *Notes on quasi-categories* [3]: we factorise the straightening–unstraightening adjunction as the composite of three adjunctions (in fact, two adjunctions and one equivalence), each of which we show to be a Quillen equivalence. One of these adjunctions (the equivalence) is easily seen to be a Quillen equivalence (in fact, an equivalence of model categories). To prove that the remaining two adjunctions are Quillen equivalences, I will use my recent proof of Joyal's Cylinder Conjecture [1].

References

[1] Alexander Campbell. Joyal's cylinder conjecture. arXiv:1911.02631, 2019.

[2] Gijs Heuts and Ieke Moerdijk. Left fibrations and homotopy colimits II. arXiv:1602.01274, 2016.

[3] André Joyal. Notes on quasi-categories. `https://www.math.uchicago.edu/~may/IMA/Joyal.pdf`, 2008.

[4] Jacob Lurie. *Higher topos theory.* Annals of Mathematics Studies, vol. 170, Princeton University Press, Princeton, NJ, 2009.

[5] Danny Stevenson. Covariant model structures and simplicial localization. *North-West. Eur. J. Math.* **3** (2017), 141–203.